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ELSEVIER

15 December 1994

OPTICS
COMMUNICATIONS

Optics Communications 113 (1994) 9–14

Optical optimization of binary phase fan-out elements

K.J. Weible, H.P. Herzig

Institute of Microtechnology, University of Neuchâtel, CH-2000 Neuchâtel, Switzerland

Received 16 June 1994

Abstract

An optical parallel processor is presented which optimizes binary phase diffractive optical elements using simulated annealing. A liquid crystal television is used to simulate the phase element. Experimental results are presented for the optimization of fan-out elements. The performance of the optical parallel processor is analyzed using Fast Fourier Transform methods.

1. Introduction

Binary phase fan-out elements, often referred to as Dammann gratings, produce a two-dimensional array of beams. In some cases the multiple beams have to be uniform in intensity, while in others a varied distribution of intensities is desired. In all cases, these elements need to be efficient, so to avoid system losses.

There are essentially two main approaches for the optimization of phase fan-out elements, either parametric optimization or by iterative Fourier techniques [1]. The iterative Fourier techniques do not lend themselves to optical implementation since they involve the recognition and manipulation of the amplitude and phase in both the object and Fourier domains. However, a parametric approach can be readily implemented by an optical parallel processing system.

Powerful computers are usually needed in the parametric optimization of phase fan-out elements due to the time consuming computations involved. In many cases a simulated annealing algorithm is employed to find a highly efficient design, and a downhill algorithm is then used to improve the uniformity of the optimized diffraction orders [2,3]. While the simulated annealing algorithm is very effective in avoiding local minima, it requires many iterations and is very time consuming. Thus, once a design with high efficiency has been achieved a downhill algorithm is then employed to further improve the uniformity of the optimized diffraction orders.

An optical parallel processor can be exploited to realize the Fourier transform operation during the simulated annealing stage of the optimization procedure. Besides providing an essentially instantaneous Fourier transform, the optical processor is also able to handle design problems where a Fast Fourier Transform (FFT) implementation is not applicable, e.g. transmission through complex amplitude media. The optical parallel processor is realized using a liquid crystal television (LCTV) to actively modulate the phase of individual pixels within the base unit cell of the diffractive element. Once the element has been optimized by the optical processor for high efficiency, a downhill algorithm on a mainframe computer is employed to improve uniformity, as before. This last stage also serves to remove any noise introduced during the optical optimization.

2. Phase modulation using a LCTV

The spatial light modulator which we are using is one of three LCTVs originally employed in a video

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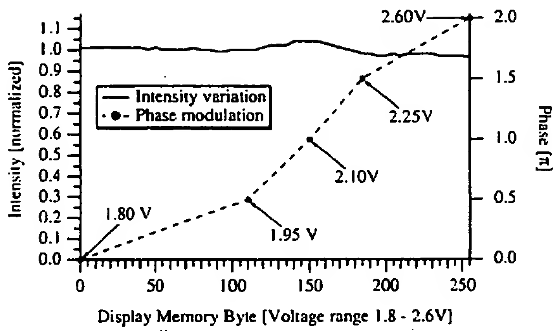


Fig. 1. Measured phase shift and intensity variation versus addressing voltage of the LCTV. The addressing voltage does not vary linearly with the display memory byte.

projector. The electronics of the projector are still used to control the television, while the video signal is generated by a personal computer (PC) using a frame grabber board. These LCTVs are designed to perform as intensity modulators by effectively providing a 90° rotation of the input polarization. With these devices both a very high contrast ratio and good gray scale performance are possible [4]. In our optimization of binary phase elements we desire to modulate the phase of the impinging beam without modifying its polarization. This is possible if the LCTV is operated at a lower voltage range and the polarization of the impinging beam is parallel to the one of the principal axes of the index ellipsoid of the liquid crystal [5]. Originally the displays are driven by an AC signal with an amplitude varying between 2.9–4.2 Volt. By reducing the amplitude of the drive voltage down to the range 1.8–2.6 Volt, we are able to achieve 2π phase modulation. For an input polarization of $-7°$, the output polarization remains very stable at 60° over the full phase modulation range. The phase of each pixel may be independently modulated between 0 and 2π, while the amplitude transmittance is only slightly modulated, p-p variation of 8.3%, Fig. 1. Furthermore, since we are simulating binary phase elements, typically we are switching the phase between 0 and π. The intensity transmission of the LCTV is essentially the same for these two phase values.

The ability of the LCTV to modulate phase is shown using a Mach–Zehnder interferometer. The LCTV is inserted in one of the arms of the interferometer and a rectangular region of the LCTV is shifted in phase by π, Fig. 2. For the simulation of the phase element to be sufficiently precise, the uniformity of the LCTV must be of high quality. A phase variation of approximately 1 fringe ($\lambda = 488$ nm) has been measured across the entire LCTV surface. Since we are using only a portion of the screen, the phase variation across the unit cell is even less than 2π. The phase variation across the display is mostly linear, which produces only a slight translation of the Fourier transform in the detector plane. Since the detector of the optical processor is aligned with respect to the beam, this translation effect is insignificant. The magnitude of higher order phase variations are negligible. Thus, the LCTV is perfectly suited to function as an electrically-addressable phase modulator.

3. Optimization with an optical parallel processor

The phase fan-out elements are a two-dimensional repetition of a base unit cell. The Fourier transform of these elements produces a two-dimensional array of delta functions. The amplitude of the delta functions depends on the phase structure of the unit cell, whereas their lattice spacing is determined by the size of the unit cell. In the optimization of the fan-out element, the individual pixels of the base unit cell are modified to find the phase structure that produces the desired intensity distribution in the Fourier transform.

The optimization of the unit cell of the phase fan-out element is realized using a relatively simple optical parallel processing system, Fig. 3. The most important element is the LCTV, which has been discussed in detail above. In a similar application these LCTVs have been demonstrated as efficient programmable phase kinoforms [6]. The Fourier transform of the phase

Fig. 2. LCTV in a Mach–Zehnder interferometer displaying π phase modulation.

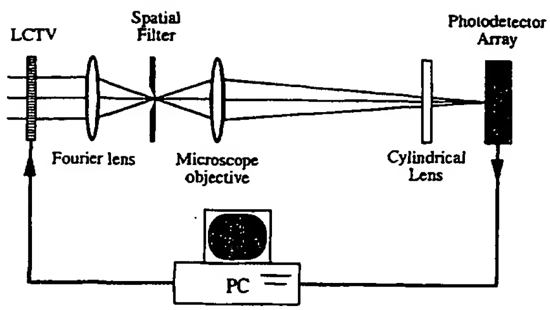


Fig. 3. Schematic of the optical parallel processor. The Fourier lens, $f=380$ mm, transforms the information displayed on the LCTV. The microscope objective images the Fourier plane onto the photodetector array with about $50\times$ magnification. The cylindrical lens adapts the rectangular diffraction pattern, due to the asymmetric spacing of the LCTV pixels ($80\ \mu m \times 90\ \mu m$), to the square format of the photodetector array.

information encoded within the television is achieved by traversing the LCTV with a collimated laser source (Ar laser at $\lambda=488$ nm) and measuring the intensity in the focal plane of a Fourier transform lens. Also in the Fourier plane, a diffraction pattern resulting from the pixellated structure of the LCTV is observed. The zero order of this diffraction pattern is imaged onto a two-dimensional array of photodetectors using a microscope objective, while the other diffraction orders are blocked. The lateral magnification of the image produced by the microscope objective is adjusted to correctly scale the Fourier transform with respect to the photodetector array. The phase information of the Fourier transform is lost as only its intensity is being measured. Yet, for the optimization of fan-out elements, only the intensity variation of the diffraction orders is of interest and the phase information is not of concern.

To begin the design process, each pixel of the phase structure is randomly initialized to values of 0 or π. The phase value of a randomly selected pixel or a group of pixels within the phase structure is inverted from 0 to π or from π to 0. The decision whether to accept the modification or not is based on the simulated annealing Metropolis algorithm [7]. The cost function of the annealing algorithm is defined as the sum of the deviation of the measured intensities of the photodetectors for the orders to be optimized from the desired values, i.e.

$$\mathrm{Cost} = \sum_{\text{opt orders}} (I_{\text{meas}} - I_{\text{desired}})^2. \tag{1}$$

The lateral dimensions of the Fourier transform are scaled with respect to the photodetector array, so that one photodetector corresponds to one diffraction order. Thus, to optimize the intensity within a certain diffraction order, the intensity I_{meas} measured by the corresponding photodetector is incorporated into the cost function of the simulated annealing algorithm.

After completion of the simulated annealing optimization with the optical processor, the resulting unit cell is stored to be used as the starting point for further improvement on a mainframe computer or a powerful PC. The second step of the optimization process applies a downhill algorithm to improve the uniformity of the intensity distribution in the Fourier transform of the already highly efficient design. This part of the optimization procedure involves relatively few iterations and does not need the long computational time associated with the simulated annealing algorithm. The purpose of the downhill algorithm is to improve the uniformity while maintaining the high efficiency of the design. However, in our case it also serves to remove any noise from the design which is due to the imperfections of the optical implementation.

4. Experimental results

The experimental implementation of the optical processing system has limitations placed on the number of unit cells that need to be illuminated and the resolution of a unit cell, i.e. the number of pixels per unit cell. It is necessary to illuminate more than 1 unit cell so that the envelope sinc function representing each diffraction order does not significantly interfere with the neighboring order. A minimum of 2×2 unit cells must be illuminated to achieve the separation of the main lobes of the sinc functions. Better distinction of the individual orders is achieved with the illumination of 3×3 unit cells. The resolution of a unit cell is constrained by the spot size of the beam in the focal plane of the Fourier lens. The number of pixels per unit cell determines the cell length L and thus the separation of the diffraction orders in the Fourier plane, given by

$$\Delta x = \lambda f / L, \tag{2}$$

where f is the focal length of the Fourier lens. A spot size of 40 μm is measured in the focal plane of the Fourier lens, Fig. 3. As a result, the system is limited

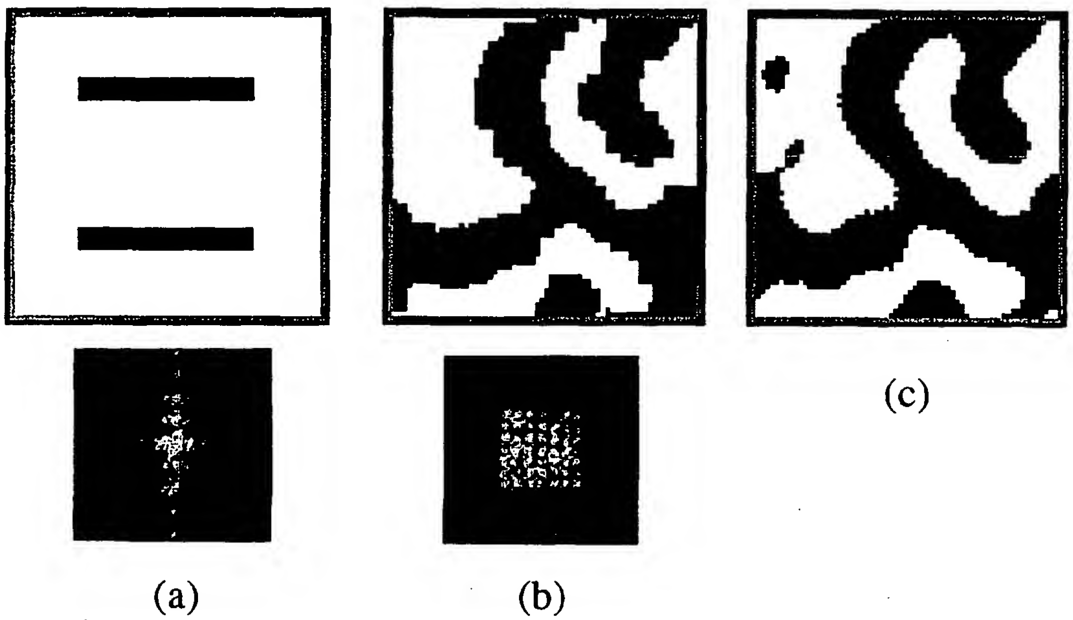


Fig. 4. Experimental results for a 64 × 64 pixel design. The top row shows the pixel distribution of the binary phase element (white = 0, black = π) and the bottom row shows the optical Fourier transformation; (a) the initial starting point, (b) the optically optimized result for 7 × 7 orders, and (c) the final result after downhill optimization on the computer.

to a resolution of 32 × 32 pixels per unit cell to insure that the neighboring diffraction orders do not overlap. A cell length of 64 pixels produces diffraction orders with only 32 μm separation, and interference between the neighboring orders is observed. Since the spot size is not diffraction limited, but is dominated by aberrations, it can be significantly decreased by improving the optical quality of the beam.

The experimental results of the simulated annealing optimization with the optical processor are presented for resolutions of 64 × 64 and 32 × 32 LCTV pixels. While the results of the 64 × 64 resolution exhibit interference noise due to the overlapping orders, the optical optimization is still capable of iterating to a good design, provided the desired intensity distribution in the diffraction orders is not too complex. The results of the 32 × 32 resolution illustrates the capabilities of the processor when the diffraction orders are well separated in the Fourier plane.

The performance of the designs are judged by their diffraction efficiency and their uniformity. The diffraction efficiency η is defined to be the percentage of the total transmitted intensity located in the optimized diffraction orders, i.e.

$$\eta = \left(\sum_{\text{opt orders}} I_{\text{order}} \right) \Big/ I_{\text{trans}}\,, \tag{3}$$

where I_{order} is the intensity in the optimized order and I_{trans} is the total transmitted intensity. The uniformity δ of the design is expressed in terms of the standard deviation of the intensities σ_I from the mean value I_{avg}. The standard deviation is normalized with respect to the mean value, thus its magnitude is expressed as a percentage of the mean value. The uniformity δ is thus given by

$$\delta = \sigma_I / I_{\text{avg}}. \tag{4}$$

4.1. 2 × 2 unit cells with 64 × 64 resolution

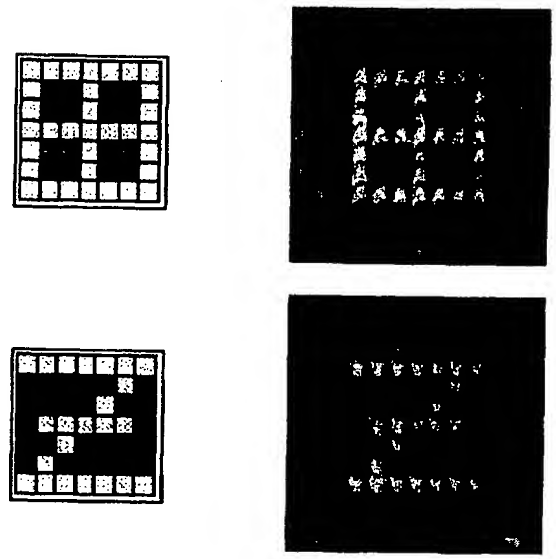
To begin the optimization procedure, the unit cell was set to the ambiguous pattern of Fig. 4a. A group of 7 × 7 pixels, centered around a randomly selected pixel in the unit cell, were modified in each of the 2 × 2 unit cells. At the completion of the simulated annealing optimization, the pattern had evolved to its final configuration, Fig. 4b. The results of the optical optimi-

zation provide a good starting point for the downhill optimization realized with FFT techniques on a computer. When analyzed using an FFT, the optically optimized result exhibited an efficiency of $\eta=77.7\%$, and a uniformity of $\delta=40.9\%$. After completing the downhill optimization of the design on the mainframe computer, the final design, Fig. 4c, was characterized by an efficiency of $\eta=77.0\%$ and a uniformity of $\delta=1.6\%$.

4.2. 6×6 unit cells with 32×32 resolution

The interference noise due to the overlapping orders is no longer present when 6×6 unit cells of 32×32 resolution are used for the optical optimization. Each of the diffraction orders is well separated from the others and is imaged onto an individual detector of the photodetector array. Consequently, the intensity distribution in the diffraction orders may be optimized by the optical system for more complex arrangements. The limit to the complexity of the desired intensity distribution is determined by the resolution of the unit cell. The orders are well separated, but the minimum feature size is severely limited in the unit cell containing only 32×32 pixels.

To illustrate the capability of the optical processor using 6×6 unit cells with a resolution of 32×32, a complex distribution of the intensity was sought in the optimized orders. The two patterns shown in Fig. 5a were used as the target values for the optimization with the optical processor. In both cases, the processor was able to optimize the design of the unit cell so as to achieve the desired intensity distribution amongst its diffraction orders, Fig. 5b. The diffraction orders are well separated and good contrast is observed between the orders optimized for high intensity and those optimized for low intensity. The optimized design for the enclosed cross pattern has an efficiency of $\eta=64.1\%$ and a uniformity of $\delta=29.1\%$. After the downhill optimization, the final design has an efficiency of $\eta=73.5\%$ and a uniformity of $\delta=1.9\%$. The optimized design for the Z pattern has a similar performance with an efficiency of $\eta=64.7\%$ and a uniformity of $\delta=29.4\%$ before and $\eta=72.2\%$ and $\delta=1.6\%$ after downhill optimization.



(a) (b)

Fig. 5. Experimental results for a 32×32 pixel design; (a) the desired intensity distribution amongst the 7×7 orders, (b) the results achieved by the optical processor.

5. C nclusions

The intensity distribution in the Fourier plane following the LCTV exhibits a large horizontal variation as illustrated in the photos of the experimental results. Theoretically, the intensity of the diffraction orders should be symmetric about the zero order for binary phase elements [8]. The intensity variation amongst the orders is quite large, even when a design that has been optimized on the computer to have good uniformity is used. The intensity gradient appears to be due to the refresh rate of the drive voltage of the individual pixels of the LCTV. The optimized orders exhibit poor uniformity in all of the designs realized by the optical processor. The standard deviation was always between 30%–40% of the mean value. Thus, the uniformity of the designs could not be corrected during the optical optimization, but was improved during the downhill algorithm on the computer.

The speed of the optical parallel processor is limited by the slow response time of the LCTV. The LCTV uses twisted-nematic liquid crystal displays which have a response time on the order of tens of msec. The electronics of the video projector drive the LCTV at the video rate of 60 Hz, which corresponds well with the speed of the crystals themselves. Consequently, each modification of the pixels of the LCTV takes approxi-

mately 30–50 ms. The number of modifications per iteration depends on the number of pixels per unit cell, and can range from several hundred to several thousand modifications. Thus the slow response of the LCTV has a large effect on the computation time associated with the simulated annealing optimization.

The optical parallel processor has been presented for the simulated annealing optimization of binary phase diffractive elements for fan-out applications. However, the optical processor can be applied in other areas as well, including problems where an FFT representation is not possible. Furthermore, the optimization method described is ideally suited for the design of phase-only reconfigurable diffractive elements to be realized by the LCTV. The optimized design realized during the optical optimization directly compensates the imperfect phase modulation characteristics of the LCTV. Finally, the extension of the optical processor for the optimization of multi-level phase elements is straightforward.

References

[1] O. Bryngdahl and F. Wyrowski, in: Progress in Optics XXVIII (1990) 1.
[2] M.P. Dames, R.J. Dowling, P. McKee and D. Wood, Appl. Optics 30 (1991) 2685.
[3] A. Vasara, M.R. Taghizadeh, J. Turunen, J. Westerholm, E. Noponen, H. Ichikawa, J.M. Miller, T. Jaakkola and S. Kuisma, Appl. Optics 31 (1992) 3320.
[4] K.J. Weible, N. Collings and A.R. Pourzand, Optical Memory and Neural Networks 1 (1992) 157.
[5] N. Konforti and E. Marom, Optics Lett. 13 (1988) 251.
[6] J. Amako and T. Sonehara, Appl. Optics 30 (1991) 4622.
[7] S. Kirkpatrick, C.D. Gelatt, Jr. and M.P. Vecchi, Science 220 (1983) 671.
[8] R.L. Morrison, J. Opt. Soc. Am. A 9 (1992) 464.